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INSTANTON-DRIVEN GLUON SATURATION

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We report on the interesting possibility of instanton-driven gluon saturation in lepton-nucleon scattering at small Bjorken-x. The explicitly known instanton gauge field serves as a concrete realization of an underlying non-perturbative saturation mechanism associated with strong classical fields. The conspicuous, intrinsic instanton size scale known from lattice simulations, turns out to determine the saturation scale. The "colour glass condensate" can be identified in our approach with the QCD-sphaleron state, dominating instanton-induced processes in the softer regime.

Lepton-nucleon scattering at small Bjorken-x uncovers a novel regime of QCD, where the coupling $\alpha_{\rm s}$ is (still) small, but the parton densities are so large that conventional perturbation theory ceases to be applicable. In general, one expects non-linear corrections to the evolution equations [1] to arise and to become significant in this regime, potentially taming the growth of the gluon distribution towards a "saturating" behaviour. Much interest has recently been generated through association of the saturation phenomenon with a multiparticle quantum state of high occupation numbers, the "Colour Glass Condensate" that correspondingly, can be viewed [2] as a strong *classical* colour field $\propto 1/\sqrt{\alpha_{\rm s}}$.

In this contribution, we shall briefly summarize our results on the interesting possibility of instanton-driven saturation at small Bjorken-x. Being extended non-perturbative and topologically non-trivial fluctuations of the gluon field, instantons [3] (I) represent a fundamental *non-perturbative* aspect of QCD. Notably, the functional form of the instanton gauge field $A_\mu^{(I)}$ is explicitely known and its strength is $\propto 1/\sqrt{\alpha_{\rm s}}$, just as needed. In addition, we shall summarize why an identification of the "Colour Glass Condensate" with the QCD-sphaleron state appears very suggestive [4, 5]. From I-perturbation theory we learned, that the instanton contribution tends to strongly increase towards the softer regime [6–9], were saturation effects are expected to occur. Our results crucially rely on non-perturbative information from high-quality lattice simulations [10–12]. For related approaches associating instantons with high-energy scattering, see Refs. [13, 14]. Instantons in the context of small-x saturation have also been studied recently by Shuryak

and Zahed [15], with conclusions differing in part from those of our preceeding work [4,5,16,17]. Their main emphasis rests on Wilson loop scattering, and lattice information was not used in their approach.

An investigation of saturation in $\gamma^* P$ scattering becomes most transparent in the colour-dipole picture [18]. At high energies, the lifetime of the $q\overline{q}$-dipole, into which the incoming γ^* fluctuates, is much longer than the interaction time between this $q\overline{q}$-pair and the hadron. For small $x_{\rm Bj}$, this gives rise to the familiar factorized expression of the inclusive photon-proton cross sections,

$$\sigma_{L,T}(x_{\rm Bj}, Q^2) = \int_0^1 dz \int d^2\mathbf{r}\, |\Psi_{L,T}(z,r)|^2\, \sigma_{\rm DP}(r,\ldots)\,. \tag{1}$$

Here, $|\Psi_{L,T}(z,r)|^2$ denotes the modulus squared of the (light-cone) wave function of the virtual photon, calculable in pQCD, and $\sigma_{\rm DP}(r,\ldots)$ is the $q\overline{q}$-dipole - nucleon cross section. The variables are the transverse $(q\overline{q})$-size $\mathbf{r}$ and the photon's longitudinal momentum fraction z carried by the quark. The dipole cross section includes the main non-perturbative contributions. Within pQCD [18,19], $\sigma_{\rm DP}$ is known to vanish with the area πr^2 of the $q\overline{q}$-dipole. Besides this phenomenon of "colour transparency" for small $r = |\mathbf{r}|$, the dipole cross section is expected to saturate towards a constant, once the $q\overline{q}$-separation r exceeds a certain saturation scale r_s.

In our study, the question is: Can background instantons of size $\sim \langle\rho\rangle$ give rise to a saturating form $\sigma^{(I)}_{\rm DP}(r,\ldots) \propto \langle\rho\rangle^2$ for $r \gtrsim \langle\rho\rangle$? Our strategy was to start from I-perturbation theory [7,8] in DIS, and then to achieve the desired continuation to the saturation regime with the crucial help of lattice data [10,11]. In a complementary approach, we have considered the semi-classical, non-abelian eikonal approximation. It results in the identification of the dipole with a Wilson loop, scattering in the non-perturbative colour field of the proton. The field $A^{(I)}_\mu \propto 1/\sqrt{\alpha_{\rm s}}$ due to background instantons was studied as a concrete example, leading to results in qualitative agreement with the first approach. Due to the limitation of space, we focus on the first approach only and refer to Refs. [5,20] for the second one.

Let us first consider briefly the simplest (idealized) I-induced process, $\gamma^*\, g \Rightarrow q_{\rm R}\overline{q}_{\rm R}$, with one flavour only and no final-state gluons. More details may be found in Ref. [4]. Already this simplest case illustrates transparently that in the presence of a background instanton, the dipole cross section indeed saturates with a saturation scale of the order of the average I-size $\langle\rho\rangle$. We start by recalling the results for the total $\gamma^* N$ cross section within I-perturbation theory from Ref. [7],

$$\sigma_{L,T}(x_{\rm Bj}, Q^2) = \int\limits_{x_{\rm Bj}}^{1} \frac{dx}{x}\left(\frac{x_{\rm Bj}}{x}\right) G\left(\frac{x_{\rm Bj}}{x},\mu^2\right)\int d\,t\; \frac{d\,\hat\sigma^{\gamma^* g}_{L,T}(x,t,Q^2)}{d\,t}; \tag{2}$$

$$\frac{d\hat\sigma^{\gamma^* g}_{L}}{dt} = \frac{\pi^7}{2}\frac{e_q^2}{Q^2}\frac{\alpha_{\rm em}}{\alpha_{\rm s}}\left[x(1-x)\sqrt{tu}\,\frac{\mathcal{R}(\sqrt{-t})-\mathcal{R}(Q)}{t+Q^2} - (t\leftrightarrow u)\right]^2, \tag{3}$$

with a similar expression for $d\,\hat\sigma^{\gamma^*\,g}_{T}/d\,t$. Here, $G\left(x_{\rm Bj},\mu^2\right)$ denotes the gluon density and L,T refers to longitudinal and transverse photons, respectively. Note that Eqs. (2), (3) involve the "master" integral $\mathcal{R}(\mathcal{Q})$ with dimension of a length,

$$\mathcal{R}(\mathcal{Q}) = \int_0^\infty d\rho\; D(\rho)\rho^5(\mathcal{Q}\rho){\rm K}_1(\mathcal{Q}\rho). \tag{4}$$

In usual I-perturbation theory, the ρ-dependence of the I-size distribution $D(\rho)$ in Eq.(4) is known [21] for sufficiently small ρ, $D(\rho) \approx D_{I-\mathrm{pert}}(\rho) \propto \rho^{6-\frac{2}{3}n_f}$, the power law increase of which, leads to (unphysical) IR-divergencies from large-size instantons. However, for sufficiently large virtualities $\mathcal{Q}$ in DIS , the crucial factor $(\mathcal{Q}\rho)\, K_1(\mathcal{Q}\rho) \sim e^{-\mathcal{Q}\rho}$ exponentially suppresses large size instantons and I-perturbation theory holds, as shown first in Ref. [7]. Replacing the I-size distribution in Eq. (4) with the one from the lattice $D_{\mathrm{lattice}}(\rho)$ (see Fig. 1 (left)), this restriction will not be necessary anymore, whence $\mathcal{R}(0) = \int_0^\infty d\rho\; D_{\mathrm{lattice}}(\rho)\rho^5 \approx 0.3$ fm becomes finite and a $\mathcal{Q}^2$ cut is no longer required.

By means of a change of variables and a subsequent $2d$-Fourier transformation, Eqs. (2), (3) may indeed be cast [4] into a colour-dipole form (1). We obtain e.g. for the longitudinal case, using $\hat{Q} = \sqrt{z\,(1-z)}\,Q$,

$$\left(|\Psi_L|^2\,\sigma_{\mathrm{DP}}\right)^{(I)} \approx |\;\Psi_L^{\mathrm{pQCD}}(z,r)\;|^2\;\frac{1}{\alpha_{\mathrm{s}}}\,x_{\mathrm{Bj}}\,G(x_{\mathrm{Bj}},\mu^2)\,\frac{\pi^8}{12} \tag{5}$$

$$\times\left\{\int_0^\infty d\rho D(\rho)\,\rho^5\,\left(\frac{-\frac{d}{dr^2}\left(2r^2\frac{\mathrm{K}_1(\hat{Q}\sqrt{r^2+\rho^2/z})}{\hat{Q}\sqrt{r^2+\rho^2/z}}\right)}{\mathrm{K}_0(\hat{Q}r)} - (z\leftrightarrow 1-z)\right)\right\}^2 .$$

The strong peaking of $D_{\mathrm{lattice}}(\rho)$ around $\rho \approx \langle\rho\rangle$, then implies

$$\left(|\Psi_{L,T}|^2\sigma_{\mathrm{DP}}\right)^{(I)} \Rightarrow \begin{cases} \mathcal{O}(1) \text{ but exponentially small}; & r \to 0\,,\\ |\;\Psi_{L,T}^{\mathrm{pQCD}}\;|^2\;\frac{1}{\alpha_{\mathrm{s}}}\,x_{\mathrm{Bj}}\,G(x_{\mathrm{Bj}},\mu^2)\,\frac{\pi^8}{12}\,\mathcal{R}(0)^2; & \frac{r}{\langle\rho\rangle}\gtrsim 1\,. \end{cases} \tag{6}$$

Hence, the association of the intrinsic instanton scale $\langle\rho\rangle$ with the saturation scale r_s becomes apparent from Eqs. (5), (6): $\sigma_{\mathrm{DP}}^{(I)}(r,\ldots)$ rises strongly as function of r around $r_s \approx \langle\rho\rangle$, and indeed *saturates* for $r/\langle\rho\rangle > 1$ towards a *constant geometrical limit*, proportional to the area $\pi\,\mathcal{R}(0)^2 = \pi\left(\int_0^\infty d\rho\, D_{\mathrm{lattice}}(\rho)\,\rho^5\right)^2$, subtended by the instanton. Since $\mathcal{R}(0)$ would be divergent within I-perturbation theory, the information about $D(\rho)$ from the lattice is crucial for the finiteness of the result.

Now we turn to the more realistic process with an arbitrary number of gluons and flavours in the final state, which causes a significant complication. On the other hand, it is due to the inclusion of final-state gluons that the identification of the QCD-sphaleron state with the colour glass condensate has emerged [4, 5], with the qualitative "saturation" features remaining unchanged. Most of the I-dynamics resides in the I-induced $q^*(q')\,g(p)$-subprocesses with an incoming off-mass-shell quark q^* originating from photon dissociation. The important kinematical variables are the I-subprocess energy $E = \sqrt{(q'+p)^2}$ and the quark virtuality $Q'^2 = -q'^2$.

It is convenient to account for the final-state gluons by means of the so-called "$I\bar{I}$-valley method" [22]. It allows to achieve via the optical theorem an elegant summation over the gluons in form of an exponentiation, with the effect of the gluons residing entirely in the $I\bar{I}$-valley interaction $-1 \le \Omega_{\mathrm{valley}}^{I\bar{I}}(\frac{R^2}{\rho\bar{\rho}} + \frac{\rho}{\bar{\rho}} + \frac{\bar{\rho}}{\rho}; U) \le 0\,.$ The new collective coordinate R_μ denotes the $I\bar{I}$-distance, while the matrix U characterizes the relative $I\bar{I}$-colour orientation. Most importantly, the functional form of $\Omega_{\mathrm{valley}}^{I\bar{I}}$ is analytically known [23, 24] (formally) for *all* values of $R^2/(\rho\bar{\rho})$.

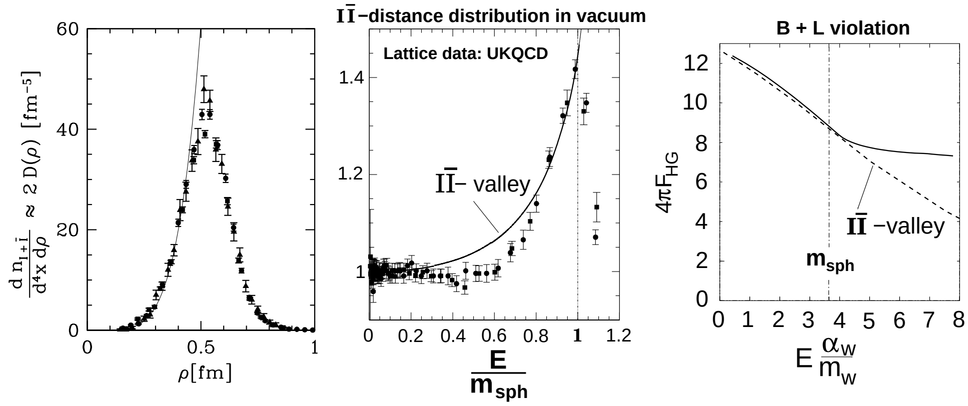

Figure 1. (Left) UKQCD lattice data [10–12] of the $(I+\bar{I})$-size distribution for quenched QCD ($n_f = 0$). Both the sharply defined I-size scale $\langle\rho\rangle \approx 0.5$ fm and the parameter-free agreement with I-perturbation theory [11, 12] (solid line) for $\rho \lesssim 0.35$ fm are apparent. (Middle) UKQCD lattice data [10, 11] of the (normalized) $I\bar{I}$-distance distribution and the corresponding $I\bar{I}$-valley prediction [4] displayed versus energy in units of the QCD sphaleron mass $m_{\rm sph}$. (Right) The same trend for electroweak $B + L$ -violation is apparent from an independent numerical simulation of the suppression exponent for two-particle collisions ('Holy Grail' function) $F_{\rm HG}(E)$ [26, 27]

Our strategy here is identical to the one for the "simplest process": Starting point is the $\gamma^* N$ cross section, this time obtained by means of the $I\bar{I}$-valley method [8]. The next step is a variable and Fourier transformation into the colour-dipole picture. The dipole cross section $\tilde{\sigma}_{\rm DP}^{(I),{\rm gluons}}(\mathbf{l}^2, x_{\rm Bj}, \ldots)$ before the final 2d-Fourier transformation $\mathbf{l} \leftrightarrow \mathbf{r}$ to the dipole size $\mathbf{r}$, arises simply as an energy integral over the I-induced total $q^* g$ cross section from Ref. [8],

$$\tilde{\sigma}_{\rm DP}^{(I),{\rm gluons}} \approx \frac{x_{\rm Bj}}{2}\, G(x_{\rm Bj}, \mu^2) \int_0^{E_{\rm max}} \frac{d\,E}{E} \left[\frac{E^4}{(E^2 + Q'^2)\, Q'^2}\, \sigma_{q^*\,g}^{(I)}\left(E, \mathbf{l}^2, \ldots\right) \right], \quad (7)$$

involving in turn integrations over the $I\bar{I}$-collective coordinates $\rho, \bar{\rho}, U$ and R_μ.

In the softer regime of interest for saturation, we again substitute $D(\rho) = D_{\rm lattice}(\rho)$, which enforces $\rho \approx \bar{\rho} \approx \langle\rho\rangle$ in the respective $\rho, \bar{\rho}$-integrals, while the integral over the $I\bar{I}$-distance R is dominated by a *saddle point*,

$$\frac{R}{\langle\rho\rangle} \approx \text{function}\left(\frac{E}{m_{\rm sph}}\right); \quad m_{\rm sph} \approx \frac{3\pi}{4}\frac{1}{\alpha_{\rm s}\,\langle\rho\rangle} = \mathcal{O}(\text{ few GeV }). \quad (8)$$

At this point, the mass $m_{\rm sph}$ of the QCD-sphaleron [6, 25], i.e the barrier height separating neighboring topologically inequivalent vacua, enters as the scale for the energy E. The saddle-point dominance implies a one-to-one relation,

$$\frac{R}{\langle\rho\rangle} \Leftrightarrow \frac{E}{m_{\rm sph}}; \quad \text{with } R = \langle\rho\rangle \Leftrightarrow E \approx m_{\rm sph}. \quad (9)$$

Our continuation to the saturation regime now involves crucial lattice information about $\Omega^{I\bar{I}}$. The relevant lattice observable is the distribution of the $I\bar{I}$-distance [4, 11] R, providing information on $\left\langle \exp[-\frac{4\pi}{\alpha_{\rm s}}\Omega^{I\bar{I}}] \right\rangle_{U,\rho,\bar{\rho}}$ in Euclidean space.

Due to the crucial saddle-point relation (8), (9), we may replace the original variable $R/\langle\rho\rangle$ by $E/m_{\rm sph}$. A comparison of the respective $I\bar{I}$-valley predictions with the UKQCD lattice data [4,10,11] versus $E/m_{\rm sph}$ is displayed in Fig. 1 (middle). It reveals the important result that the $I\bar{I}$-valley approximation is quite reliable up to $E \approx m_{\rm sph}$. Beyond this point a marked disagreement rapidly develops: While the lattice data show a *sharp peak* at $E \approx m_{\rm sph}$, the valley prediction continues to rise indefinitely for $E \gtrsim m_{\rm sph}$! It is remarkable that an extensive recent and completely independent semiclassical numerical simulation [26] shows precisely the same trend for electroweak $B+L$-violation, as displayed in Fig. 1 (right).

It is again at hand to identify $\Omega^{I\bar{I}} = \Omega^{I\bar{I}}_{\rm lattice}$ for $E \gtrsim m_{\rm sph}$. Then the integral over the I-subprocess energy spectrum (7) in the dipole cross section appears to be dominated by the sphaleron configuration at $E \approx m_{\rm sph}$ The feature of saturation analogously to the "simplest process" in Sec. 3.1 then implies the announced identification of the colour glass condensate with the QCD-sphaleron state.

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